

27 points

Hint.

- Special sequences for the case of THREE input variables A, B, C (or x, y, z):

$$A \text{ (or } x) = 00001111, B \text{ (or } y) = 00110011, C \text{ (or } z) = 01010101$$

- The truth table for the minterms (using the special sequences for x, y, z) are as follows:

Table 1

Truth tables for minterms using the special sequences for x, y, z

$xyz=00000001$	$xyz'=00000010$	$xy'z=00000100$	$xy'z'=00001000$
$x'yz=00010000$	$x'yz'=00100000$	$x'y'z=01000000$	$x'y'z'=10000000$

Tasks of Midterm I

Task 1 (4 pt). Design a three-input **minimal** AND-OR circuit L with the following truth table:

$$T = [x, y, z; L] = [00001111, 00110011, 01010101; \mathbf{01010111}]$$

Hint.

Step 1. Find an appropriate Boolean expression as a sum of minterms (that is in CSoP form). (1 pt)

Step 2. Create a Karnaugh Map based on result of the Step 1. (1 pt)

Step 3. Find an appropriate MSoP (1 pt)

Step 4. Design a three-input **minimal** AND-OR circuit L which corresponds to the Step 3 (1 pt)

Solution.

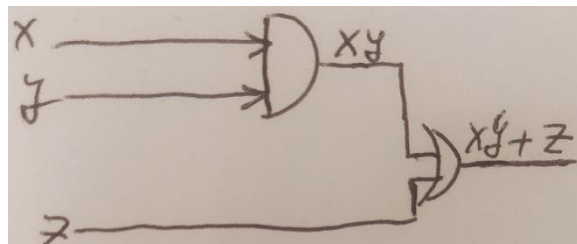
Step 1. (1 pt) Input sequences for the given circuit are special sequences. Represent the output sequence of bits generated by the given circuit, namely, **01010111** as a sum of minterms according to Table 1. We have **01010111** $= x'y'z + x'yz + xy'z + xyz' + xyz$

Step 2. (1 pt) Put checks in K-map according to the last expression.

(A)	yz	yz'	y'z'	y'z
x	V	V		V
x'	V			V

Step 3. (1 pt) MSoP = $z + xy$

Step 4. (1 pt) Minimal AND-OR circuit L which corresponds to is MSoP



Task 2 (5=3+1+1 pt).

Find the truth table $T=T(E)$ for the Boolean expression

$$E=E(x, y, z)=(xy+z')(x+y'z')'+(xy'+z)(yz')'+x'y+x'yz'$$

Hint.

Step 1. Find SoP form for E. (3 pt)

Step 2. Find CSoP form (1 pt)

Step 3. Use Table 1 to find $T(E)$ ($T(E)$ will be the sequence of eight digits 0 or 1). (1 pt)

Solution.

Step 1. (3 pt). Find SoP form for E.

$$E=E(x, y, z)=(xy+z')(x+y'z')'+(xy'+z)(yz')'+x'y+x'yz' =$$

$$[(xy+z')(x'(y+z))]+[(x'+y)z'(y'+z)]+x'y =$$

$$[xyx'y+yxy'z'+x'yz'+x'zz']+[x'y'z'+x'z'z'+yy'z'+yz'z]+x'y =$$

$$[x'yz']+[x'y'z'] + x'y = x'y'z' + x'y$$

Step 2. (1 pt). Find CSoP form for E.

$$x'y'z' + x'y = x'y'z' + x'y(z+z') = x'y'z' + x'yz+x'yz' =$$

Step 3. (1 pt). Find $T(E)$

$$T(E)=x'y'z' + x'yz+x'yz' = 10000000 + 00010000 + 00100000 = \mathbf{1011\ 0000}$$

Answer: $T(E)=1011\ 0000$

Task 3 (4 pt=1+3)

Find a Boolean expression $E=E(x, y, z)$ in **MSoP form** corresponding to the truth table $T(E)$:

$$T(E)=\mathbf{10110011}$$

Hint.

1. Find CSoP form for E by using Table 1. (1 pt)
2. Then use Karnaugh map method to reduce CSoP form to an MSoP form. It is enough to find at least one MSoP form if there are several such forms. (3 pt)

Solution.

Step 1. (1 pt). Find CSoP form for E by using Table 1.

$$CSoP(E)=x'y'z'+x'yz'+x'yz+xyz'+xyz$$

Step 2. (3 pt).

	yz	yz'	y'z'	y'z
x	V	V		
x'	V	V	V	

It is easy to see that

$$\mathbf{MSoP=y + x'z'}$$

Task 4 (8=4+4 pt)

1. Find a MSOP for the Boolean expression E given by Table (A). (Write as an expression). (4pt)

2. Find a MSOP for the Boolean expression E given by Table (B). (Write as an expression). (4pt)

(A)	zt	zt'	z't'	z't
xy		V	V	V
xy'		V	V	V
x'y'		V		
x'y	V		V	V

(B)	zt	zt'	z't'	z't
xy			V	
xy'	V	V	V	V
x'y'	V		V	
x'y	V		V	V

Solution.

MSOP = $y'zt' + x'yt + xt' + xz' + yz'$

There are many MSOP which contain 4 summands and 10 literals, for example,

MSOP = $z't' + xy' + x'yz' + x'zt$.

In all MSOP the expression $(z't' + xy')$ is a common part. It is enough to show one of possible MSOP

Task 5. (2 pt). Let S be a universal set and A and B are subsets of S. Prove De Morgan's law:

$(A \cap B)' = A' \cup B'$

Solution.

Let $x \in (A \cap B)' \Leftrightarrow x \in U - (A \cap B) \Leftrightarrow x \notin (A \cap B) \Leftrightarrow x \notin A \text{ or } x \notin B \Leftrightarrow x \in A' \text{ or } x \in B' \Leftrightarrow x \in (A' \cup B')$

Task 6 (2 pt =1+1)

Subtask 6.1 (1 pt). Find the negation of the proposition below

$\exists x \forall y \forall z P(x, y, z)$

Subtask 6.2 (1 pt). Domain of all predicates P(x) below is the set of *all integers*. What are the truth values of the corresponding propositions?

a) $\forall x (x < -55)$ (0.25 pt)

b) $\exists !x (x^3 = 1)$ (0.25 pt)

c) $\exists !x (4x^2 + 5 = 1)$ (0.25 pt)

d) $\exists x (7x = 4x + 1)$ (0.25 pt)

Solution.

Subtask 6.1 (1 pt).

$\neg(\exists x \forall y \forall z P(x, y, z)) = \forall x \neg(\forall y \forall z P(x, y, z)) = \forall x \exists y \neg(\forall z P(x, y, z)) = \forall x \exists y \exists z (\neg P(x, y, z))$

Subtask 6.2 (1 pt).

a) False

b) True

c) False

d) False

Task 7 (2 pt). Prove that set of all points of the plane with rational coordinates is countable.

Proof.

The set of all point in task, say set A, is: $A = \{(x, y) \in \mathbb{Q} \times \mathbb{Q} \mid \mathbb{Q} \text{ is the set of rational numbers}\}$.
Another way to describe A is:

$$A = \bigcup_{x \in \mathbb{Q}} \left(\bigcup_{y \in \mathbb{Q}} (x, y) \right).$$

Since $\mathbb{Q}$ is countable, then for each chosen $x \in \mathbb{Q}$ the set $B_x = \bigcup_{y \in \mathbb{Q}} (x, y)$ can be considered as a copy of $\mathbb{Q}$, that B_x is in bijection with $\mathbb{Q}$. Then $A = \bigcup_{x \in \mathbb{Q}} B_x$ and all B_x are nonempty and disjoint. Hence the family $\{B_x \mid x \in \mathbb{Q}\}$ forms a partition of A and amount of such B_x is countable, then A is a countable union of countable subsets. Hence, A is countable.

Note. A student may submit another proof. But it must be “proof” that is not an explanation.

Good Luck!

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