

ADA University, SITE, Fall 2024, Discrete Structures
Midterm 1 (October 19, 2022). **SOLUTION SHEET**
27 points

Full Name: _____ **CRN:** _____

- Special sequences for the case of THREE input variables A, B, C (or x, y, z):
 $A \text{ (or } x) = 00001111, B \text{ (or } y) = 00110011, C \text{ (or } z) = 01010101$
- The truth table for the minterms (using the special sequences for x, y, z) are as follows:

Table 1

Truth tables for minterms using the special sequences for x, y, z

$xyz=00000001$	$xyz'=00000010$	$xy'z=00000100$	$xy'z'=00001000$
$x'yz=00010000$	$x'yz'=00100000$	$x'y'z=01000000$	$x'y'z'=10000000$

Tasks of Midterm I

Task 1 (4 pt). Design a three-input **minimal** AND-OR circuit L with the following truth table:

$$T = [x, y, z; L] = [00001111, 00110011, 01010101; \mathbf{01011011}]$$

- Step 1. Write truth table as a sum of minterms. (1 pt)
 Step 2. Create a Karnaugh Map based on result of the Step 1. (1 pt)
 Step 3. Find an appropriate MSOP (1 pt)
 Step 4. Design a three-input **minimal** AND-OR circuit L which corresponds to the Step 3 (1 pt)

Solution.

Step 1. (1 pt) Input sequences for the given circuit are special sequences. Represent the output sequence of bits generated by the given circuit, namely, **01011011** as a sum of minterms according to the Table 1. We have **01011011** = $x'y'z + x'yz + xy'z' + xyz' + xyz$

Step 2. (1 pt) Put checks in K-map according to the last expression.

(A)	yz	yz'	y'z'	y'z
x	V	V	V	
x'	V			V

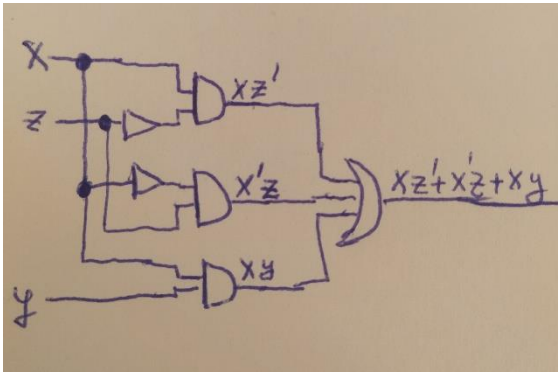
Step 3. (1 pt)

MSOP = $\mathbf{xz' + x'z} + xy$ **OR** MSOP = $\mathbf{xz' + x'z} + yz$

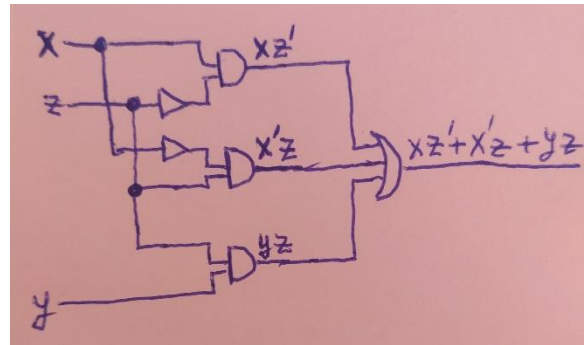
(It will be enough if the student suggests only one option).

Step 4. (1 pt) Minimal AND-OR circuit L which corresponds to is MSOP

(See next page)



$$\text{MSOP} = \underline{xz'} + \underline{x'z} + xy$$



$$\text{MSOP} = \underline{xz'} + \underline{x'z} + yz$$

(It will be enough if the student suggests only one option).

Task 2 (5=3+1+1 pt).

Find the truth table $T=T(E)$ for the Boolean expression

$$E = E(x, y, z) = ((x+y)+z)' (x'+yz')' + (xy+z)'(yz') + x'z + x'yz'$$

Hint.

Step 1. Find SoP form for E. (3 pt)

Step 2. Find CSoP form (1 pt)

Step 3. Use Table 1 to find $T(E)$ ($T(E)$ will be the sequence of eight digits 0 or 1). (1 pt)

Solution.

Step 1. (3 pt). Find SoP form for E.

$$\begin{aligned} E = E(x, y, z) &= ((x+y)+z)' (x'+yz')' + (xy+z)'(yz') + x'z + x'yz' = \\ &= [(x+y)z'] [x(y'+z)] + [(xy)'z] (yz') + x'z + x'yz' = \\ &= [\underline{xz'} + \underline{yz'}] [\underline{xy'} + \underline{xz}] + [\underline{x'z} + \underline{y'z}] (yz') + x'z + x'yz' = \\ &= \underline{xz'} \underline{xy'} + \underline{xz'} \underline{xz} + \underline{yz'} \underline{xy'} + \underline{yz'} \underline{xz} + \underline{x'z} yz' + \underline{y'z} yz' + x'z + x'yz' = \\ &= xy'z' + x'z + x'yz' \end{aligned}$$

Step 2. (1 pt). Find CSoP form for E.

$$E(x, y, z) = xy'z' + x'z + x'yz' = xy'z' + x'yz + x'y'z + x'yz'$$

Step 3. (1 pt). Find $T(E)$

$$T(E) = xy'z' + x'yz + x'y'z + x'yz' = 0000 \ 1000 + 0001 \ 0000 + 0100 \ 0000 + 0010 \ 0000 = \mathbf{0111 \ 1000}$$

Answer: $T(E) = \mathbf{0111 \ 1000}$

Task 3 (4 pt=1+3)

Find a Boolean expression $E=E(x, y, z)$ in **MSoP form** corresponding to the truth table $T(E)$:

$$T(E) = \mathbf{01011011}$$

- Find CSoP form for E by using Table 1. (1 pt)

2. Then use Karnaugh map method to reduce CSOP form to an MSOP form. It is enough to find at least one MSOP form if there are several such forms. (3 pt)

Solution.

Step 1. (1 pt). Find CSOP form for E by using Table 1.

$$\text{CSOP}(E) = x'y'z + x'yz + xy'z' + xyz' + xyz$$

Step 2. (3 pt).

It is easy to see that

$$\text{MSOP} = xz' + x'z + xy \quad \text{or}$$

$$\text{MSOP} = xz' + x'z + yz$$

	yz	yz'	y'z'	y'z
x	V	V	V	
x'	V			V

(It will be enough if the student suggests only one option).

Task 4 (8=4+4 pt)

1. Find a MSOP for the Boolean expression E given by Table (A). (Write as an expression). (3pt)

2. Find a MSOP for the Boolean expression E given by Table (B). (Write as an expression). (3pt)

(A)	zt	zt'	z't'	z't
xy		V	V	V
xy'	V		V	V
x'y'	V	V		V
x'y			V	V

(B)	zt	zt'	z't'	z't
xy	V		V	V
xy'	V	V	V	V
x'y'				
x'y	V		V	V

Solution.

For the case (A)

$$\text{MSOP (A)} = xz' + yz' + z't + y't + x'y'z + xyt'$$

For the case (B)

$$\text{MSOP (B)} = xy' + yz' + yt$$

Task 5 (3 pt)

Subtask 5.1 (1 pt). Find the negation of the proposition below

$$\forall x \exists y \exists z P(x, y, z)$$

Subtask 5.2 (2 pt). Domain of all predicates $P(x)$ below is the set of *all integers*. What are the truth values of the corresponding propositions?

- a) $\forall x(x \text{ is odd})$ (0.5 pt) b) $\exists !x(x^2=4)$ (0.5 pt)
c) $\exists !x(x^2=-4)$ (0.5 pt) d) $\exists x(x=6x+1)$ (0.5 pt)

Reminder. Quantifier $\exists !$ means “There exists only one”.

Solution.

Subtask 5.1 (1 pt).

$$\begin{aligned}\neg(\forall x \exists y \exists z P(x, y, z)) &= \exists x \neg(\exists y \exists z P(x, y, z)) = \exists x \forall y \neg(\exists z P(x, y, z)) = \\ &= \exists x \forall y \forall z (\neg P(x, y, z))\end{aligned}$$

Subtask 5.2 (2 pt).

- a) False b) False c) False d) False

Task 6 (2 pt). Prove that the set of all rational intervals on the straight-line $\mathbf{R}$ is countable. (An interval $I \subset \mathbf{R}$ is called rational if its end points are rational numbers).

Proof. (One of possible proofs). The set of all rational intervals on the straight-line $\mathbf{R}$ is the following set, say $A = \{(r_1, r_2) \mid r_1 < r_2 \text{ and } r_1, r_2 \text{ are rationals}\}$.

Now for each chosen rational number r_1 we consider the family $A(r_1) = \{(r_1, r_2) \mid r_1 < r_2\}$. Clear that set of all r_2 in definition of $A(r_1)$ is countable subset of the set of rationals. Hence, $A(r_1)$ is a countable family.

Since r_1 is rational number so we have countable set of families $A(r_1)$ each of which is countable itself.

Finally, the seeking set $A = \bigcup_{(\text{over all } r_1)} A(r_1)$. That is the set A is a countable union of countable families and therefore A is countable set. Proved.

Task 7 (1 pt). Let S be a universal set and A and B are subsets of S . Prove De Morgan's law:

$$(A \cap B)' = A' \cup B'$$

Proof. Let $x \in (A \cap B)' \Leftrightarrow x \in S - (A \cap B) \Leftrightarrow x \notin (A \cap B) \Leftrightarrow (x \notin A \text{ or } x \notin B) \Leftrightarrow (x \in A' \text{ or } x \in B') \Leftrightarrow x \in (A' \cup B')$ ■

Good Luck!

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