

ADA University, SITE, Fall 2023, Discrete Structures/Fundamentals of Mathematics
Midterm 1 (October 14, 2022).

SOLUTION SHEET

25 points

Full Name: _____

CRN: _____

Hint.

- Special sequences for the case of THREE input variables A, B, C (or x, y, z):
 $A \text{ (or } x) = 00001111, B \text{ (or } y) = 00110011, C \text{ (or } z) = 01010101$
- The truth table for the minterms (using the special sequences for x, y, z) are as follows:

Table 1

Truth tables for minterms using the special sequences for x, y, z

xyz=00000001	xyz'=00000010	xy'z=00000100	xy'z'=00001000
x'yz=00010000	x'yz'=00100000	x'y'z=01000000	x'y'z'=10000000

Tasks of Midterm I

Task 1 (4 pt). Design a three-input **minimal** AND-OR circuit L with the following truth table:

$$T = [x, y, z; L] = [00001111, 00110011, 01010101; \mathbf{11011001}]$$

Hint.

- Step 1. Write truth table as a sum of minterms. (1 pt)
 Step 2. Create a Karnaugh Map based on result of the Step 1. (1 pt)
 Step 3. Find an appropriate MSoP (1 pt)
 Step 4. Design a three-input **minimal** AND-OR circuit L which corresponds to the Step 3 (1 pt)

Solution.

Step 1. (1 pt) Input sequences for the given circuit are special sequences. Represent the output sequence of bits generated by the given circuit, namely, **11011001** as a sum of minterms according to the Table 1. We have **11011001** = $x'y'z' + x'y'z + x'yz + xy'z' + xyz$

Step 2. (1 pt) Put checks in K-map according to the last expression.

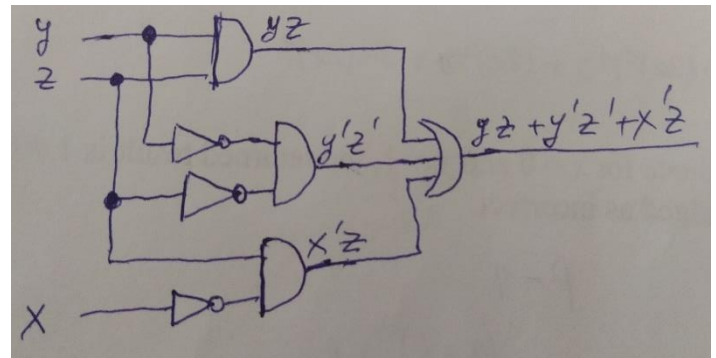
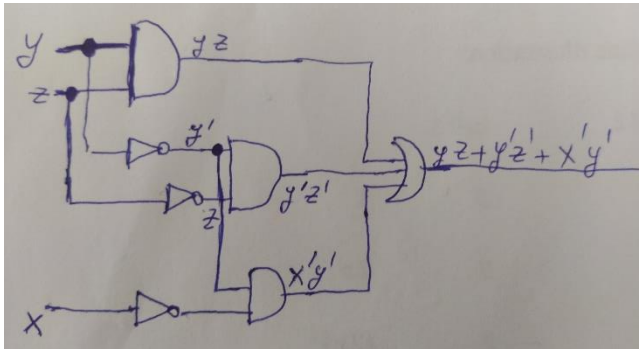
(A)	yz	yz'	y'z'	y'z
x	V		V	
x'	V		V	V

Step 3. (1 pt) MSoP = $yz + y'z' + x'y'$ or MSoP = $yz + y'z' + x'z$

Step 4. (1 pt) Minimal AND-OR circuit L which corresponds to is MSoP

$$\text{MSoP} = yz + y'z' + x'y'$$

$$\text{MSoP} = yz + y'z' + x'z$$



Task 2 (5=3+1+1 pt).

Find the truth table $T=T(E)$ for the Boolean expression

$$E = E(x, y, z) = (xy' + z')'(x + yz') + (x'z + y)'(y'z) + x'z + xy'z'$$

Hint.

Step 1. Find SoP form for E. (3 pt)

Step 2. Find CSoP form (1 pt)

Step 3. Use Table 1 to find $T(E)$ ($T(E)$ will be the sequence of eight digits 0 or 1). (1 pt)

Solution.

Step 1. (3 pt). Find SoP form for E.

$$E = E(x, y, z) = (xy' + z')'(x + yz') + (x'z + y)'(y'z) + x'z + xy'z' =$$

$$[(xy')'z''(x + yz')] + [(x'z)'y'(y'z)] + x'z + xy'z' =$$

$$[(x' + y)z(x + yz')] + [(x + z')y'(y'z)] + x'z + xy'z' =$$

$$[x'zx + x'zyz' + xyz + yzyz'] + [xy'y'z + z'y'y'z] + x'z + xy'z' =$$

$$[xyz] + [xy'z] + x'z + xy'z' = xyz + xy'z + x'yz + x'y'z + xy'z'$$

Step 2. (1 pt). Find CSoP form for E.

$$xyz + xy'z + x'yz + x'y'z + xy'z' = xyz + xy'z + x'yz + x'y'z + xy'z'$$

Step 3. (1 pt). Find $T(E)$

$$T(E) = xyz + xy'z + x'yz + x'y'z + xy'z' = 0000\ 0001 + 0000\ 0100 + 0001\ 0000 + 0100\ 0000 + 0000\ 1000 \\ = 0101\ 1101$$

Answer: $T(E) = 0101\ 1101$

Task 3 (4 pt=1+3)

Find a Boolean expression $E=E(x, y, z)$ in **MSoP form** corresponding to the truth table $T(E)$:

$T(E)=11011001$

Hint.

1. Find CSoP form for E by using Table 1. (1 pt)
2. Then use Karnaugh map method to reduce CSoP form to an MSoP form. It is enough to find at least one MSoP form if there are several such forms. (3 pt)

Solution.

Step 1. (1 pt). Find CSoP form for E by using Table 1.

CSoP(E)= $x'y'z'+x'y'z+x'yz+xy'z'+xyz$

Step 2. (3 pt).

	yz	yz'	y'z'	y'z
x	V		V	
x'	V		V	V

It is easy to see that MSoP= $yz+y'z'+x'y'$ or MSoP= $yz+y'z'+x'z$

Task 4 (6=3+3 pt)

1.Find a MSoP for the Boolean expression E given by Table (A). (Write as an expression). (3pt)

2.Find a MSoP for the Boolean expression E given by Table (B). (Write as an expression). (3pt)

(A)	zt	zt'	z't'	z't
xy			V	V
xy'	V	V	V	
x'y'		V		
x'y	V		V	V

(B)	zt	zt'	z't'	z't
xy	V		V	V
xy'	V	V	V	V
x'y'			V	
x'y	V		V	V

Solution.

MSoP= $yz'+y'zt'+x'yt+xy'z+xy't'$

MSoP= $yz'+y'zt'+x'yt+xy'z+xz't'$

MSoP= $yt+xy'+z't'$

PS. It is enough to offer one of MSoP's.

Task 5 (4 pt) Use Prim's algorithm to find a minimum spanning tree in the graph.

	<i>a</i>	2	<i>b</i>	4	<i>c</i>	2	<i>d</i>
3			2		4		1
<i>e</i>		1	<i>f</i>	3	<i>g</i>	2	<i>h</i>
1			3		1		5
	<i>i</i>	2	<i>j</i>	3	<i>k</i>	1	<i>m</i>

Solution.

One of possible solutions presented in table below

Choice	Edge	Weight
1	<i>ef</i>	1
2	<i>ei</i>	1
3	<i>ij</i>	2
4	<i>fb</i>	2
5	<i>ba</i>	2
6	<i>fg</i>	3
7	<i>gk</i>	1
8	<i>km</i>	1
9	<i>gh</i>	2
10	<i>hd</i>	1
11	<i>dc</i>	2

Since number of vertices is 12, hence number of edges in a minimal spanning tree is 11 and any spanning tree does not contain cycles.

Total weight for any minimal Spanning tree is equal to 18

Total weight=18

Task 6 (2 pt =1+1)

Subtask 6.1 (1 pt). Find the negation of the proposition below

$$\exists x \forall y \forall z P(x, y, z)$$

Solution.

$$\neg(\exists x \forall y \forall z P(x, y, z)) = \forall x \neg(\forall y \forall z P(x, y, z)) = \forall x \exists y \neg(\forall z P(x, y, z)) = \forall x \exists y \exists z (\neg P(x, y, z))$$

Subtask 6.2 (1 pt). Domain of all predicates $P(x)$ below is the set of *all integers*. What are the truth values of the corresponding propositions?

a) $\forall x (x < 1)$ (0.25 pt)

b) $\exists ! x (x^5 = -1)$ (0.25 pt)

c) $\exists ! x (2x + 3 = 1)$ (0.25 pt)

d) $\exists x (x = 4x + 1)$ (0.25 pt)

Solution.

a) False b) True c) True d) False

Task 7 (2 pt). It is known that the union of finite or countable number of countable sets is countable. Based on this result prove that the set A of all polynomials with rational coefficients is countable.

Proof. Let Q be the set of all rational numbers. A set P of all polynomials with rational coefficients is a disjoint union of the of subsets A_n , where A_n is the subset of all polynomials of degree equal n, that is,

$A_n = \{a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0 \mid a_i \in Q, i=0, 1, 2, \dots, n; a_n \neq 0\}$, $n > 0$; and $A_0 = Q$ (that is A_0 is countable).

In each A_n , $n > 0$, we have:

Under chosen $a_n, a_{n-1}, \dots, a_1$ the number of polynomials is countable (only a_0 is changes in Q);

Now we choose $a_n, a_{n-1}, \dots, a_2$ and change a_1 in Q. For each a_1 number of choices for a_0 is countable. Hence, if we change a_1 and a_0 in Q then number of corresponding polynomials is the countable union of countable number polynomials and therefore is countable. By similar reasons we obtain that A_n is countable. Finally, since $P = \sum_{n=0}^{\infty} A_n$, so P is countable.

Good Luck!

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