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- Calculus 2 / Quiz 2 / Fall, 2026
- **Student :**
- Total marks : 3 pts
- Duration : 10 minutes
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Exercise 1: [3 pts] Find the radius of convergence and interval of convergence of the following power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n 2^n}$$

Solution:

$$\begin{aligned} L &= \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| \\ \left| \frac{a_{n+1}}{a_n} \right| &= \left| \frac{(-1)^{n+1} x^{n+1}}{(n+1) 2^{n+1}} \cdot \frac{n 2^n}{(-1)^n x^n} \right| \\ \left| \frac{a_{n+1}}{a_n} \right| &= \left| \frac{x^{n+1}}{x^n} \cdot \frac{2^n}{2^{n+1}} \cdot \frac{n}{n+1} \right| = \left| x \cdot \frac{1}{2} \cdot \frac{n}{n+1} \right| = \frac{|x|}{2} \cdot \frac{n}{n+1} \end{aligned}$$

Take the limit as $n \rightarrow \infty$:

$$L = \lim_{n \rightarrow \infty} \left(\frac{|x|}{2} \cdot \frac{n}{n+1} \right) = \frac{|x|}{2} \cdot 1 = \frac{|x|}{2}$$

By the Ratio Test, the series converges absolutely when $L < 1$:

$$\frac{|x|}{2} < 1 \implies |x| < 2$$

This gives:

$$-2 < x < 2$$

Thus, the **radius of convergence** is:

$$R = 2$$

Substitute $x = -2$ into the original series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n (-2)^n}{n 2^n} = \sum_{n=1}^{\infty} \frac{(-1)^n (-1)^n 2^n}{n 2^n} = \sum_{n=1}^{\infty} \frac{(-1)^{2n}}{n} = \sum_{n=1}^{\infty} \frac{1}{n}$$

This is the **harmonic series**, which **diverges**.

Substitute $x = 2$ into the original series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n 2^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

This is the **alternating harmonic series**. Applying the Alternating Series Test:

- (1) $b_n = \frac{1}{n} > 0$
- (2) $\frac{1}{n+1} \leq \frac{1}{n}$ for all $n \geq 1$ (decreasing)
- (3) $\lim_{n \rightarrow \infty} \frac{1}{n} = 0$

Since all conditions are met, the series **converges** at $x = 2$.

- **Radius of Convergence:** $R = 2$
- **Interval of Convergence:** $(-2, 2]$ (or $-2 < x \leq 2$)