

Quiz 1 – Variant A

Full name: _____

CRN number: _____

Instructions. Show all necessary work and clearly justify your answers. Solutions should be complete and well written.

1. Determine whether the sequence

$$a_n = n \left(\sqrt{n^2 + 4} - n \right)$$

converges or diverges. If it converges, find its limit. Your solution should justify any algebraic transformation used.

2. Determine whether the series

$$\sum_{n=2}^{\infty} \frac{\ln n}{n^{3/2}}$$

converges or diverges. Choose an appropriate test and justify why the test applies.

Quiz 1 – Variant B

Full name: _____

CRN number: _____

Instructions. Show all necessary work and clearly justify your answers. Solutions should be complete and well written.

1. Determine whether the sequence

$$a_n = n \left(\sqrt{n^2 + 9} - n \right)$$

converges or diverges. If it converges, find its limit. Your solution should justify any algebraic transformation used.

2. Determine whether the series

$$\sum_{n=2}^{\infty} \frac{\ln n}{n^{4/3}}$$

converges or diverges. Choose an appropriate test and justify why the test applies.

Quiz 1 – Variant A

Full name: _____

CRN number: _____

Instructions. Show all necessary work and clearly justify your answers. Solutions should be complete and well written.

1. Determine whether the series

$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$$

converges or diverges. Choose an appropriate test and justify why the test applies.

2. Determine whether the series

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$$

converges or diverges. Select an efficient test and show all essential limit calculations.

Quiz 1 – Variant B

Full name: _____

CRN number: _____

Instructions. Show all necessary work and clearly justify your answers. Solutions should be complete and well written.

1. Determine whether the series

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^{2/3}}$$

converges or diverges. Select an appropriate convergence test, verify its hypotheses when required, and show the essential calculation.

2. Determine whether the series

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n+1)!}$$

converges or diverges. Choose an appropriate test and justify why the test applies.

Quiz 1 – Variant A

Full name: _____

CRN number: _____

Instructions. Show all necessary work and clearly justify your answers. Solutions should be complete and well written.

1. Determine whether the series

$$\sum_{n=1}^{\infty} \frac{\sqrt{n^2 + 1}}{n^2 + 3n + 1}$$

converges or diverges. Choose a suitable comparison series and justify your conclusion rigorously.

2. Determine whether the series

$$\sum_{n=1}^{\infty} \left(\frac{3n + 2}{4n + 1} \right)^n$$

converges or diverges. Choose an appropriate test and justify why the test applies.

Quiz 1 – Variant B

Full name: _____

CRN number: _____

Instructions. Show all necessary work and clearly justify your answers. Solutions should be complete and well written.

1. Determine whether the series

$$\sum_{n=1}^{\infty} \frac{\sqrt{n^2 + 4}}{n^2 + n + 2}$$

converges or diverges. Choose a suitable comparison series and justify your conclusion rigorously.

2. Determine whether the series

$$\sum_{n=1}^{\infty} \left(\frac{2n+3}{3n+4} \right)^n$$

converges or diverges. Choose an appropriate test and justify why the test applies.

Quiz 1 – Variant A – Solutions

1. Rationalizing,

$$a_n = \frac{4n}{\sqrt{n^2 + 4} + n} = \frac{4}{\sqrt{1 + 4/n^2} + 1} \longrightarrow 2.$$

Thus $\boxed{a_n \rightarrow 2}$.

2. Use the Integral Test with $f(x) = \ln x/x^{3/2}$. It is positive, continuous, and eventually decreasing. Integration by parts gives

$$\int_2^\infty \frac{\ln x}{x^{3/2}} dx = \left[-\frac{2 \ln x}{\sqrt{x}} - \frac{4}{\sqrt{x}} \right]_2^\infty = \frac{2 \ln 2 + 4}{\sqrt{2}} < \infty.$$

Hence

$$\boxed{\sum_{n=2}^\infty \frac{\ln n}{n^{3/2}} \text{ converges.}}$$

Quiz 1 – Variant B – Solutions

1. Rationalizing,

$$a_n = n \frac{9}{\sqrt{n^2 + 9} + n} = \frac{9}{\sqrt{1 + 9/n^2} + 1} \longrightarrow \frac{9}{2}.$$

Thus $\boxed{a_n \rightarrow 9/2}$.

2. Use the Integral Test with $f(x) = \ln x / x^{4/3}$, which is positive, continuous, and eventually decreasing. Integration by parts gives

$$\int \ln x \, x^{-4/3} \, dx = -3x^{-1/3} \ln x - 9x^{-1/3} + C.$$

Therefore

$$\int_2^\infty \frac{\ln x}{x^{4/3}} \, dx = \frac{3 \ln 2 + 9}{2^{1/3}} < \infty.$$

Hence

$$\boxed{\sum_{n=2}^\infty \frac{\ln n}{n^{4/3}} \text{ converges.}}$$

Quiz 1 – Variant A – Solutions

1. With $f(x) = 1/(x\sqrt{\ln x})$, the Integral Test applies. Put $u = \ln x$:

$$\int_2^\infty \frac{dx}{x\sqrt{\ln x}} = \int_{\ln 2}^\infty u^{-1/2} du = \infty.$$

Thus the series diverges.

2. Let $a_n = (n!)^2/(2n)!$. Then

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)^2}{(2n+2)(2n+1)} = \frac{n+1}{2(2n+1)} \longrightarrow \frac{1}{4} < 1.$$

By the Ratio Test, the series converges.

Quiz 1 – Variant B – Solutions

1. Use the Integral Test with

$$f(x) = \frac{1}{x(\ln x)^{2/3}}.$$

It is positive, continuous, and decreasing for $x > 1$. With $u = \ln x$,

$$\int_2^\infty \frac{dx}{x(\ln x)^{2/3}} = \int_{\ln 2}^\infty u^{-2/3} du = \left[3u^{1/3} \right]_{\ln 2}^\infty = \infty.$$

Hence the series diverges.

2. Let $a_n = (n!)^2 / (2n+1)!$. Then

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)^2}{(2n+3)(2n+2)} = \frac{n+1}{2(2n+3)} \rightarrow \frac{1}{4} < 1.$$

By the Ratio Test, the series converges.

Quiz 1 – Variant A – Solutions

1. Compare with $b_n = 1/n$:

$$\lim_{n \rightarrow \infty} \frac{\frac{\sqrt{n^2+1}}{n^2+3n+1}}{1/n} = \lim_{n \rightarrow \infty} \frac{\sqrt{1+1/n^2}}{1+3/n+1/n^2} = 1.$$

Since $\sum 1/n$ diverges, the given series diverges.

2. By the Root Test,

$$\lim_{n \rightarrow \infty} \sqrt[n]{\left(\frac{3n+2}{4n+1}\right)^n} = \lim_{n \rightarrow \infty} \frac{3n+2}{4n+1} = \frac{3}{4} < 1.$$

Thus the series converges.

Quiz 1 – Variant B – Solutions

1. Compare with $b_n = 1/n$:

$$\lim_{n \rightarrow \infty} \frac{\frac{\sqrt{n^2+4}}{n^2+n+2}}{1/n} = \lim_{n \rightarrow \infty} \frac{\sqrt{1+4/n^2}}{1+1/n+2/n^2} = 1.$$

Since the harmonic series diverges, the given series diverges.

2. By the Root Test,

$$\lim_{n \rightarrow \infty} \sqrt[n]{\left(\frac{2n+3}{3n+4}\right)^n} = \lim_{n \rightarrow \infty} \frac{2n+3}{3n+4} = \frac{2}{3} < 1.$$

Thus the series converges.